Quantum Interference to Measure Spacetime Curvature: A Proposed Experiment at the Intersection of Quantum Mechanics and General Relativity

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Abstract

An experiment in Low Earth Orbit (LEO) is proposed to measure components of the Riemann curvature tensor using atom interferometry. We show that the difference in the quantum phase $\Delta\phi$ of an atom that can travel along two intersecting geodesics is given by $mR_{0i0j}/\hbar$ times the spacetime volume contained within the geodesics. Our expression for $\Delta\phi$ also holds for gravitational waves in the long wavelength limit.

Keywords: Riemann Curvature Tensor, Atom Interferometry

In general relativity no "gravitational force" exists [1]; there is only geometry. Thus, in the absence of any forces, a test particle will undergo free fall, and will travel along a geodesic determined by the local spacetime geometry. What we normally associate with the "force of gravity" on a particle in Newtonian mechanics is not a force at all: The particle is simply traveling along the "straightest" possible path in a curved spacetime.

Nowhere is this basic difference between Newtonian gravity and general relativity more apparent than for tidal forces. Consider twin astronauts in separate, but nearby, orbits around the Earth. In Newtonian gravity the astronauts accelerate towards or away from one another due to the *difference* in the Earth's gravity between them. In general relativity, by contrast, because both astronauts do not feel any forces, they undergo free fall; they travel along geodesics in a spacetime curved by the Earth. Because the two geodesics are slightly different, one of the astronauts will see his twin drift toward or away from him. The *rate* of change of the distance X^μ separating the two astronauts depends on the local curvature tensor $R_{\mu\nu\alpha\beta}$ *along* X^μ. If $R_{\mu\nu\alpha\beta}$ changes slowly along X^μ, X^μ satisfies the geodesic deviation equation [1]; if $R_{\mu\nu\alpha\beta}$ changes appreciably along X^μ, its evolution depends on the *integral* of $R_{\mu\nu\alpha\beta}$ along X^μ [2]. In either case, one astronaut interprets the shift in his twin's position as being due to a fictitious Newtonian "gravitational tidal force" acting on his twin, even though the latter feels no forces.

The twins age differently due to their differing gravitational red shifts: The twin in the lower orbit ages less than the farther twin. However, in this general relativistic twin paradox, *both* twins feel no forces, unlike in the special relativistic case. Their age difference cannot arise within a Newtonian framework, where time is absolute. However, at the quantum level, this *age* difference for interfering atoms becomes a measureable *phase* difference directly related to the curvature.

In this Gravity Research Foundation essay, we propose an experiment in LEO to measure components of the Riemann curvature tensor. In a quantum extension of the above example, this experiment will test quantum mechanically the tenant of general relativity that local geometry determines motion. However, instead of relying on *classical* test particles, we make use of a quantum phenomenon: the fringe shifts of an interference pattern caused by phase differences of *quantum* test particles. Consequently, this experiment will probe the intersection of general relativity and quantum mechanics [3].

The conceptual roots of this proposed experiment are well established [2, 4, 5]. Its

technical roots lie in Chu's [6] and Kasevich's [7] work on atom interferometry in their measurements of the local acceleration due to gravity g using Mach-Zehnder-type interferometers [8, 9, 10, 11, 12]. An accuracy based on Newtonian dynamics of $\Delta g/g \sim 10^{-8}$ for a single measurement, and up to 10^{-11} after a two-day integration, was achieved. In these measurements, cesium atoms were thrown upwards in the Earth's gravitational field. STIRAP (Stimulated Raman Processes) were used as beam-splitters, and the relative phase of the atom traveling along the two arms of the interferometer was measured. Because the interferometer was fixed to the Earth, this phase difference is proportional to g. We propose a similar type of experiment, but now in LEO where the interferometer is in free fall [13]. The term linear in g now disappears, and the phase difference will be proportional to the Riemann curvature tensor inside the area encircled by two geodesics.

Consider an atom interferometer in LEO at a distance $R^i(t)$ from the center of the Earth. We choose a local coordinate system X^μ fixed on the center-of-mass (CM) of the apparatus. The signature of $g_{\mu\nu}$ is $(-1, 1, 1, 1)$; in linearized gravity $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Note also that $|X^i| \ll |R^i|$; we expand $h_{\mu\nu}$ about $X^i = 0$.

We then release an atom traveling along the orbit of the CM with geodesic γ_1 (see Fig. 1). At t_A, STIRAP is used to coherently split the atomic beam. One possible geodesic for the atom is still γ_1; the other corresponds to a *different* geodesic γ_2. Note that the atom behaves quantum mechanically, and that the superposition principle holds. It is *not* possible to determine which geodesic any individual atom will take. Because the spatial projection of both γ_1 and γ_2 correspond to LEOs, they will intersect with one another again after a time T as shown in Fig. 2. Detectors can then be used after coherent recombination at t_B to determine the interference pattern, and thus the phase shift $\Delta\phi$ that the atom picks up between the two possible geodesics. The loss of fringe visibility would be a measure of decoherence of the atom. Importantly, the combined spacetime path $\gamma = \gamma_1 \cup \gamma_2$ is closed (see Fig. 1), and forms the boundary of a spacetime surface $\mathcal{D}$. If the coherence time of the atom is too short to allow the atomic beam to recombine due to drift, additional STIRAP beam splitters can be used to force recombination. However, let us assume that the coherence time is long enough to allow Fig. 2.

The Schrödinger equation for the atom is

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + i\hbar N_i\nabla^i\psi - mN_0\psi, \tag{1}$$

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where ψ is the wavefunction, m is the mass of the atom, and using standard methods,

$$N_0 = \frac{1}{4}X^i X^j \partial_i \partial_j h_{00}(R(t)) + \frac{1}{2}X^j X^k \frac{dR^i}{dt}\partial_j \partial_k h_{0i}(R(t)),$$

$$N_i = X^j \partial_j h_{0i}(R(t)) - \frac{1}{2}X^j X^k \partial_j \partial_k h_{0i}(R(t)). \tag{2}$$

Our results here hold only for stationary metrics. However, from [2] and [14] our result for $\Delta\phi$ given below holds as long as the local curvature varies slowly in X^μ. From [2], $N_\mu = (N_0, N_i)$ is the four-velocity field acting on the test particle induced by the tidal field as seen by an observer at the CM.

An atom forms a wave-packet that propagates along either γ_1 or γ_2. Taking the eikonal approximation $\psi = e^{imS/\hbar}\psi_0$, where ψ_0 is the solution of eq. (1) in the absence of (N_0, N_i),

$$\frac{\partial S}{\partial t} = -\frac{1}{2}|\nabla S|^2 + N_i \nabla^i S + N_0, \qquad 0 = \frac{\psi_0}{2}\nabla^2 S + (\nabla_i S - N_i)\nabla^i \psi_0. \tag{3}$$

From eq. (2), $\nabla_i N^i = 0$. Thus, eq. (3) becomes

$$\frac{\partial S}{\partial t} = N_0, \qquad \nabla_i S = N_i, \tag{4}$$

neglecting terms of $\mathcal{O}(N^2)$. Solving eq. (4)

$$S(X) = \int_0^{X^\mu} N_\mu d\tilde{X}^\mu, \tag{5}$$

integrated along γ_1 or γ_2. The exponentiated form of eq. (5) is Yang's nonintegrable phase factor [15], but with N_μ instead of the vector potential A_μ. N_μ *does* pay a similar role in general relativity as the gauge field arising from local Galilean invariance in the nonrelativistic limit [2]. Thus,

$$\Delta\phi = \frac{m}{\hbar}\int_{\gamma_2} N_\mu d\tilde{X}^\mu - \frac{m}{\hbar}\int_{\gamma_1} N_\mu d\tilde{X}^\mu = \frac{m}{\hbar}\int_{\mathcal{D}} R_{0i0j}(R(t))\tilde{X}^i \, d\tilde{t} \, d\tilde{X}^j \tag{6}$$

by Stokes' theorem. Since the normal to $\mathcal{D}$ is a spacelike vector,

$$\Delta\phi \approx \frac{m}{\hbar}|R_{0i0j}|\mathcal{A}T \tag{7}$$

for R_{0i0j} varying slowly in time. $\mathcal{A}$ is the area contained in the two intersecting orbits in Fig. 2, and T is the transit time. Notice that while $\Delta\phi$ is proportional to R_{0i0j} and is thus independent of the choice of frame in the nonrelativisitc limit, there is an unexpected dependence of $\Delta\phi$ of a test particle on its mass. For a cesium atom in a LEO of 480 km the phase difference is $\Delta\phi_{Earth} \approx (mGM_\oplus/R^3\hbar)\mathcal{A}T \approx 0.26 \ \mathcal{A}T$ rad cm^{-2} s^{-1}.

The phase difference is proportional to the area inside the arms of the interferometer. Ultimately, this area is limited by the coherence time of the atoms. Using the parameters in [6] $T = 0.16$ s and $\mathcal{A} = 0.01$ cm^2, we get $\Delta\phi_{Earth} \approx 4\text{x}10^{-4}$ rad. A single-measurement fringe resolution ≈ 0.074 rad has been demonstrated in Earth-based experiments [6], and after one-minute's integration time, this resolution improves to 0.011 rad. Measurement of $\Delta\phi_{Earth}$ requires the fringe sensitivity be increased by a factor of 27 beyond the above. However, $\Delta\phi_{Earth} \sim T^3$ ($\mathcal{A} \sim (vT)^2$ where v is the velocity of the cesium atom after STIRAP), and increasing the coherence time T from 0.16 s to 0.5 s will bring $\Delta\phi_{Earth}$ within the sensitivity of current interferometers.

That our proposed experiment is within the realm of feasibility is due to the advances in atom interferometry. These interferometers are so sensitive because beams of atoms are used instead of beams of photons. At 100 GeV/c^2, the rest-mass of an atom is larger than the effective gravitational mass of a typical photon of 1 eV/c^2 by eleven orders of magnitude. New interferometers based on Bose-Einstein Condensates (BECs) in development [16, 17] can increase the sensitivity of atom-based interferometry significantly. In-situ phase measurements of BECs [18, 19, 20] offer the potential of even more sensitive measurements of curvature. Moreover, with long coherence times, vortices in BECs and superfluids may allow sensitive interference measurements of curvature.

Gravitational waves from astrophysical sources in the tens of kilohertz range would cause a phase shift four orders of magnitude smaller than $\Delta\phi_{Earth}$. This could in principle be detected with future improvements in atom interferometry.

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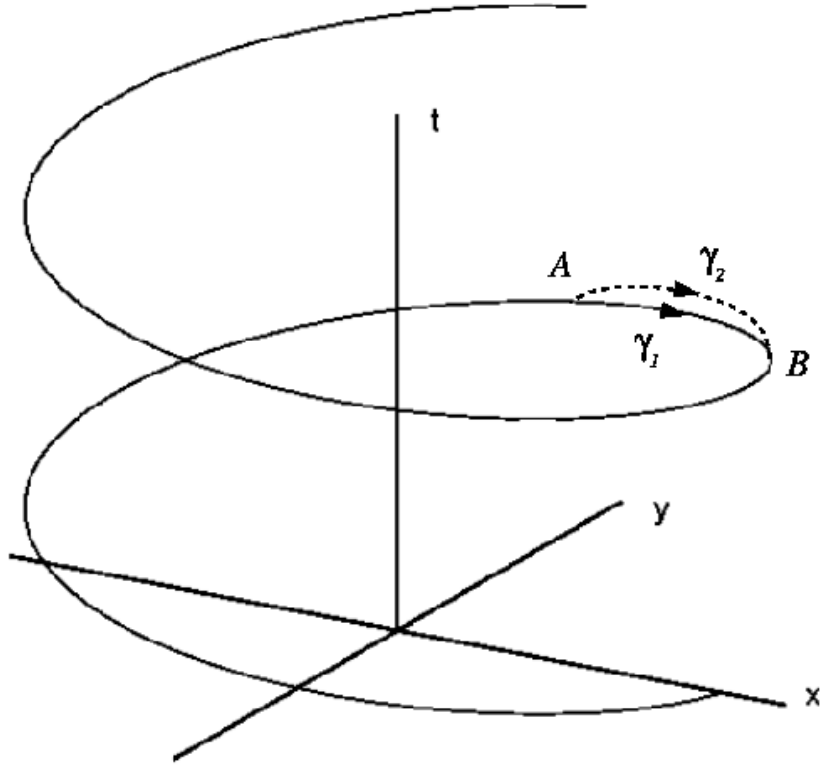
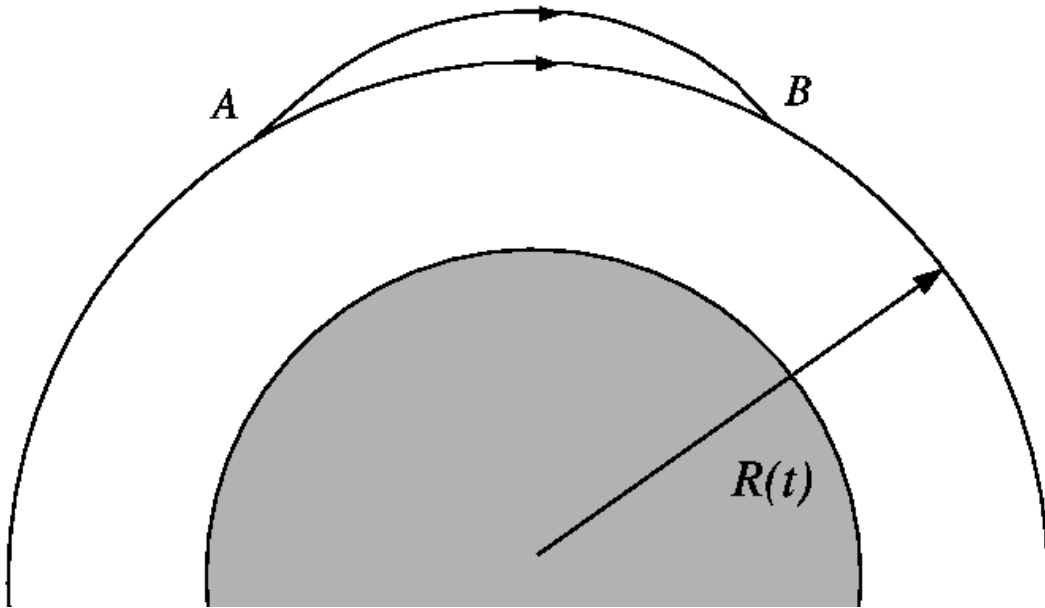FIG. 1: Sketch of the the spacetime diagram of the interferometry measurement.

FIG. 2: Sketch of the atomic CM orbit around the Earth. An atomic beam is coherently split at A, and recombined coherently at B.